Calculus and Real Analysis

Functions of One Real Variable: limit, continuity, intermediate value property, differentiation, Rolle's Theorem, mean value theorem, L'Hospital rule, Taylor's theorem, Taylor's series, maxima and minima, Riemann integration (definite integrals and their properties), fundamental theorem of calculus.

Functions of Several Real Variables: limit, continuity, partial derivatives, total derivative, maxima and minima.

Integral Calculus: double and triple integrals, change of order of integration, calculating surface areas and volumes using double integrals, calculating volumes using triple integrals.

Sequences and Series of Real Numbers: convergence of sequences, bounded and monotone sequences, Cauchy sequences, Bolzano-Weierstrass theorem, absolute convergence, tests of convergence for series – comparison test, ratio test, root test; Power series (of one real variable), radius and interval of convergence, term-wise differentiation and integration of power series.

Topology

Metric spaces: Metrics, open and closed sets, continuity, compactness, completeness.

Algebra

Matrices: systems of linear equations, rank, nullity, rank-nullity theorem, inverse, determinant, eigenvalues, eigenvectors.

Finite Dimensional Vector Spaces: linear independence of vectors, basis, dimension, linear transformations, matrix representation, range space, null space, rank-nullity theorem, orthogonality and inner products.

Groups: cyclic groups, abelian groups, non-abelian groups, permutation groups, normal subgroups, quotient groups, Lagrange's theorem for finite groups, group homomorphisms.

Rings and Modules: Definition of rings, integral domain, PID, UFD, polynomial rings, ideals, prime ideals, maximal ideals, quotient rings, Chinese remainder theorem, modules over commutative rings, submodules, quotients.

Discrete Mathematics

Counting and Combinatorics: Permutations and combinations of multisets, Binomial identities, Set partitions, Bell, Catalan, and Stirling numbers, The Pigeonhole Principle (strong form), The Inclusion and Exclusion Method and applications, Mobius Inversion, Recurrence Relations and Generating Functions.

Graph theory: the basics: graphs, paths and cycles, connectivity, trees and forests, bipartite graphs, contraction and minors, Euler tours, Hamilton Cycle. Matching and Covers: Maximum bipartite matching algorithms, Konig's Theorem, Independent Set. Cuts and Connectivity: 2-connected Graphs, Planar

Graphs: drawing, Euler's formula, Kuratowski's theorem, plane duality. Coloring: coloring maps and planar graphs, coloring vertices, coloring edges. Cayley graph, Spectrum of a graph.

Probability and Differential Equations

Probability: Random experiments, events, conditional probability, independence, Bayes theorem, random variables, mathematical expectation, variance, moments, Chebyshev's inequality, Law of large numbers, Central Limit Theorem.

Differential Equations: Bernoulli’s equation, exact differential equations, integrating factors, integral curves, homogeneous differential equations, method of separation of variables, linear differential equations of second order with constant coefficients, method of variation of parameters, Cauchy-Euler equation.